Investment Management Association

Research Paper

Modelling Income Drawdown Strategies

Edd Denbee

Contents

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Abstract

We build upon existing work on income drawdown approaches by analysing a number of strategies, some new. There is a particular focus on variable percentage strategies operating according to a pre-determined withdrawal rule.

In our analysis, we firstly examine income streams. In addition to modelling the median outcome for each strategy, we present the 10^{th} and 90^{th} percentile outcomes to gauge the extent of the uncertainty inherent in income drawdown.

We then quantify both the pay-out phase and the bequest potential in Money's Worth terms to allow for comparison with annuity products and between the drawdown strategies modelled. This section also includes sensitivity analysis of the impact of charges and of different asset allocation.

The paper does not reach firm conclusions regarding the desirability of one drawdown strategy over another, nor does it suggest that drawdown is inherently superior to annuity products. Rather, it attempts to present a range of possibilities on which to base a wider debate on the future of retirement income.

Introduction

There have been many papers written about income drawdown schemes as an alternative method of pension decumulation, focusing on different forms of periodic withdrawal from an individual's accrued funds.[1] Dus, Maurer and Mitchell present and compare a number of withdrawal strategies.[2] Drawing extensively upon their work and work by Peter Brady,[3] we build upon the existing literature by exploring a number of potential drawdown strategies, particularly those based on variable percentages. Alongside median outcomes, we also present 10th and 90th percentile figures to give an idea of the risk/reward trade-off inherent in income drawdown. In addition, by combining expected payouts with expected bequests, we are able to show the total amount generated by such a set of investments and not solely the income stream.

The paper presents models of the following different strategies:

- A fixed value per year;
- A fixed percentage per year;
- A variable percentage per year (with four different approaches);
- A 1/E(t) approach related to an individual's life expectancy.

Within the literature, there are a number of other strategies, the most prevalent of which is the so-called 1/T rule where T is a maximum age less the current age.[4] We have not modelled this as it results in minimal income in the early years of retirement (when T is very large and thus 1/T small) combined with a high level income stream towards the latter part of retirement. This is in contrast to expected consumption patterns which, for a healthy individual, tend to drop with increasing age throughout retirement.[5]

The analysis is in three sections:

- The first explains the modelling technique and outlines the assumptions that go into the model;
- The second summarises the different withdrawal strategies that we considered;
- Finally, we look at these income streams with quantitative eyes calculating the Money's Worth values and median bequests. We combine these, taking into account a potential tax charge and compare the total returns with that of an annuity.

[1] See for example Milevsky and Robinson (2000); Dus et al. (2003), Blake et al. (2003), Albrecht and Maurer (2003).

[2] Dus et al. (2003).

[3] See Brady (2007).

[4] See Horneff et al. (2006) and Dus et al. (2003).

[5] On retirement consumption patterns, see Banks et al. (2004), particularly Figure 9.1 on p.247. Clearly, there may be particular individuals within a cohort who find themselves needing a different income profile: for example, those requiring residential health care later in life. Depending upon the interaction with future welfare state and local authority provision, these people may require a considerably different consumption pattern. However, the decision process in such situations is likely to be complex in that it may well not be clear at the point of retirement what future health care needs will be.

1. The Model

We start with a pension holder who has reached retirement aged 65 and has accumulated assets to turn into an income stream during their retirement. We model a range of different approaches to withdrawal and compare them to a standard annuity purchase based upon current market rates.

For the purposes of this work, we consider an annuity to be an index-linked annuity, which although not the most popular form of annuity, is most suitable for comparison with a real income stream from a diversified portfolio given the uncertainty around future inflation.[6] In order to make a comparison with the relevant annuity rate, our pension holder is a 65 year old, non-smoking man with a private pension fund of £25,000. He takes a single life annuity with no guarantee. This is approximately the median fund size that is used to purchase an annuity.[7] We take the mean of the annuity rate values quoted by the FSA Annuity Comparative Tables.[8] As of 3rd October 2007, the mean price was an annual real income of 4.66% of the initial annuity size. Although it is true that the Open Market Option allows pension holders to seek out the best annuity price on the market, only about a third of people actually change providers, suggesting that the mean price is adequate.[9]

The model that we present is a stochastic process whereby 10,000 possible experiences for each withdrawal strategy are computed and the median and particular percentiles are shown. We start by setting a particular asset mix for the portfolio of 60% equities, 20% bonds and 20% cash products.

Looking across a range of simulated portfolios, we determined that this portfolio represented a trade-off between risk and return that would make it adequate for the purpose of a drawdown solution.[10] Each year, four things happen to the portfolio:

- It is affected by investment performance which increases or decreases the value of the fund;
- A withdrawal is made to provide the retiree with an income. The withdrawal always occurs at the beginning of each year;
- It is subject to an annual management charge of 1%;
- It is rebalanced to maintain the 60:20:20 asset split.

We consider periods of 45 years, taking a 65 year old pension holder to 110 years old. The chance of the pension holder being alive decreases with each year. The graphical representation of income streams is limited to the first 35 years of retirement as the chance of being alive past 100 is very low. For all other calculations, e.g. Money's Worth and bequest potential, we calculate values based on the full 45 year retirement period.

[6] See, for example, Harrison et al. (2006) and Stark (2002). We also ran the model in nominal terms using the relevant returns data. However, the nominal returns data has a built in historical inflation figure of approximately 4%. We decided that to compare an annuity, which is priced based upon long term inflation forecasts which are considerably less than 4%, with a nominal income drawdown income flow would not be a consistent approach. In fact this approach would unfairly disadvantage the annuity with respect to income drawdown.

[7] Former Economic Secretary Ed Balls, Hansard, 4 July 2006, Column 727.

[8] http://www.fsa.gov.uk/tables/

[9] HM Treasury (2006). See Table 4.1 p.24.

[10] See Sensitivity Section for more detail about this issue.

Investment Performance

We use a Monte Carlo simulation that selects an annual return, at random, from a normal distribution profile of returns. These profiles are based upon the historical experience of UK equity, gilt and t-bill returns over the period 1899-2001.[11] We calculate the simple average annual return, the standard deviation of these one-year returns, together with the geometric mean annual return. We run the simulation entirely in real terms thereby matching the income stream to that of the index-linked annuity. Table 1 below shows the real returns and standard deviations.

Table 1: Real Returns and Standard Deviations (1899-2001)

Type of Investment	Simple Average Rate of Return	Standard Deviation	Geometric Mean Rate of Return
Equity	7.23%	20.24%	5.37%
Bond	1.98%	14.13%	1.06%
T-Bills	1.18%	6.68%	0.98%

Source: Barclays Capital Equity Gilt Study

We are modelling payouts and investment returns that occur annually. The means and the standard deviations that we use, therefore, must correspond to this approach. Consequently, we input the simple average and standard deviation of the annual returns and not the geometric mean. As we would expect, after running the Monte Carlo simulations we find that, despite only inputting the simple average annual return with the corresponding standard deviation, the simple average of the geometric means after 45 years closely resembles the observed historical geometric mean.[12] This is illustrated in Table 2.

Table 2: Geometric Mean Rates of Return

Type of Investment	Simulated Geometric Mean Rate of Return	Actual Geometric Mean Rate of Return
Equity	5.33%	5.37%
Bond	1.00%	1.06%
T-Bill	0.96%	0.98%

Source: Barclays Capital Equity Gilt Study and author's calculations

We also find that the standard deviation of these geometric means is considerably lower than that of the annual returns. Again, this is to be expected, as standard deviations tend to decrease with increased holding periods.

Investment returns for equities, bonds and treasury bills are not completely independent. According to Markowitz's Portfolio Selection Theory, we need to consider a portfolio as an entire entity and not a collection of independent investments.[13] To this end we can calculate a 3-asset portfolio mean and standard deviation in the following way:

$$R_P = aR_A + bR_B + cR_C$$
$$s.t: a + b + c = 1$$
$$\sigma_P^2 = a^2\sigma_A^2 + b^2\sigma_B^2 + c^2\sigma_C^2 + 2ab\sigma_A\sigma_B\rho_{AB} + 2ac\sigma_A\sigma_C\rho_{AC} + 2bc\sigma_B\sigma_C\rho_{BC}$$

Where R = mean rate of return, a = % equity, b = % bond, c = % cash, σ = standard deviation and ρ = correlation between assets.

By numerically calculating these values, we can create a normally distributed returns profile based upon our portfolio and the experienced returns history from 1899-2001. The Monte Carlo simulation selects a return, at random, from this Gaussian distribution. This results in a random walk, with a drift determined by the mean values, through the realm of possible returns.

[11] Barclays Capital Equity Gilt Study, 2002.

[12] This issue is discussed in Brady (2007) p.29, footnote 46.

[13] For a good summary see http://home.cc.umanitoba.ca/~grantc/61354/Markowitz%20Portfolio%20Selection.pdf or Markowitz (1952).

Presentation of Simulation Results

In the simulation results, we present the median of the simulations along with the 10th (corresponding to sustained exceptionally bad investment returns over a 45 year period) and 90th (sustained exceptionally good investment returns) percentiles.

There is an inherent problem with plotting percentiles in a simulation such as this: every year, an individual simulation will change rank, so when we plot a percentile we invent an experience that has not necessarily taken place. To avoid this, and get a more consistent experience, we need to find a way of ranking all the profiles. We do this using the concept of Money's Worth (see Section Three).

However, the profile with a particular percentile's Money's Worth value may be a rather unusual looking profile that is not really typical of the other profiles. To address this issue, we create a given percentile by averaging the 500 profiles either side of this value to smooth the uncertainty and give us a profile that resembles the percentile that we want.

We note that the average of the results, and so the expected profile, is actually considerably higher than the median. This is due to the fact that the cumulative effect of a run of positive investment returns takes an individual further from the original value than the cumulative effect of a run of negative investment returns and so weights the average towards the high end. In fact, the expected profile would lie halfway between the 10th and 90th percentiles.

For this reason, we also note that the difference between the 90th percentile (resulting from a run of excellent returns) and the median is considerably larger than the difference between the 10th percentile (resulting from a run of bad returns) and the median. The upside benefit, in monetary terms, is therefore greater than the downside risk. Considering a concave utility function of wealth whereby a drop in wealth loses more utility than the equivalent increase in wealth would gain, we can say that a system whereby the upside benefit is greater than the downside risk is a more suitable risk to take than one where the upside and downside are the same.

For a profile to be worse than the 10th percentile it requires either an exceptionally bad sustained run of returns, or a type of market crash at the beginning of the fund's life that leaves little chance of recovery due to withdrawals while the fund value is low. Similarly, for a profile to be better than the 90th percentile requires a sustained run of exceptionally good returns, particularly in the early years where the fund size is larger, and the chance of being alive higher.

An assumption built into the simulation that each year's return is independent of previous returns means that very extreme results are more likely in the simulation than in reality (in other words, we have not modelled mean reversion). In this respect we considered what would have happened had a pension holder experienced the sequences of returns that occurred during the 20th century. We modelled 86 periods; where we were able, these were complete 45 year periods.

The period of poor returns running up to the First World War would have resulted in a number of years whereby an income drawdown scheme would have produced returns of the magnitude of the 10th percentile of simulated returns. It is worth noting that in those periods, pre-1917 where the returns were extremely bad, the worst returns never dropped more than 6% below the 10th percentile. However, we considered 68 periods from 1917 onwards, some of which are incomplete. The Money's Worth value cannot decrease as you consider a longer period. For example, for a 30 year period that has a Money's Worth value greater than the 10th percentile the corresponding entire 45 year period will do too. We see that the number of periods in which a pension holder would have had returns worse than the 10th percentile is no greater than one (except in the fixed value strategy where up to three periods would have given an income stream worse than the 10th percentile). This is illustrated in Table 3.

Table 3: Number of periods in which returns would have been more extreme than the 10th and 90th percentiles.

Withdrawal Strategy	Number of times less than 10th Percentile	Number of times less than 10th Percentile exc. pre-1917	Number of times greater than 90th Percentile
Fixed Value 4.66p per £1	12	2	10
Fixed Value 5.59p per £1	12	3	2
Fixed Percentage 4.66%	3	0	4
Fixed Percentage 5.59%	6	1	4
Linearly Increasing Percentages	6	1	7
Exponentially Increasing Percentages	3	0	3
Equivalent Annuity Percentages	4	0	2
Canadian RRIF	8	1	4
1/E(t) - Annuitants	4	0	3

2. Simulation Results

In this section we present the findings across the different strategies modelled:

- A fixed value per year;
- A fixed percentage per year;
- A variable percentage per year (with four different approaches);
- A 1/E(t) approach related to an individual's life expectancy.

In the following charts, both the income streams and median pot size are presented on the same chart. The left hand axis represents the value of the income stream per £1 of initial pot size. So, for example, if the value aged 65 is 4.66, an income of 4.66p per £1 can be expected. Hence, an individual with an initial pot of £100,000 would receive a payment of £4,660. The median, 10th and 90th percentiles all correspond to this axis. The right hand axis measures the remaining pot size. This is measured as a value per £1 of initial pot size. For example, a value of 91 means that 91p per £1 of the initial pot size remains. The individual who started with £100,000 would have £91,000 remaining.

Fixed Value

In this approach, a fixed amount is withdrawn each year mimicking the annuitisation process. Like an annuity, this has the advantage of giving an individual knowledge of what their pension will be in any given year, with two additional potential benefits: the potential to withdraw larger amounts per year than a standard annuity; and the potential for a bequest.

However, unlike an annuity, the individual doesn't benefit from mortality credits and carries the risk of running out of money. The moment at which this happens, if it happens at all, depends upon three things: the amount that is withdrawn each year; the investment performance that occurs during the lifetime of the fund; and how long an individual lives.

We considered withdrawal rates beginning at age 65 of both 4.66p per £1, based upon the index-linked annuity rate, and 5.59p per £1, 120% of the annuity rate. Under current income drawdown rules an individual is allowed to withdraw a maximum of 120% of the Government Actuary's Department's quoted annuity rate. This is expressed in nominal terms and is currently of the order of 8.5% for a 65 year old male. As our calculations are presented in real, and not nominal, terms we use 120% of the mean index-linked annuity rate as a proxy for this maximum. Clearly, given current life expectancy changes, the higher withdrawal strategy carries a high degree of risk. However, even the lower withdrawal rate presents what is probably an unacceptable level of risk. See Table 4.

Table 4: Fixed Value (Real Terms) Withdrawals and Life of Fund

Withdrawal amount (beginning at age 65)	Age at which funds run out		
	10th Percentile	Median	90th Percentile
4.66p per £1	83	98	>110
5.59p per £1	80	90	>110

Fixed Percentage

The risk of running out of money could be addressed by withdrawing a fixed percentage of the remaining assets each year, rather than a fixed amount, thus having a constant benefit-wealth ratio:

$$B_t = \omega V_t$$

Where B_t is the benefit received in year t after retirement, ω is the fixed percentage and V_t is the size of the pension fund in year t.

In this case, by definition, there is never a chance of running out of money completely. The danger is that with a decreasing fund size the amount of income that an individual can withdraw will decrease with time. We set the percentages to match an index-linked annuity rate (4.66%) and 120% of the annuity rate (5.59%) in the first year:

- We find that for the 4.66% withdrawal (see Figure 1), the median income stream very gently falls. This is due to the fact that the expected returns after management charges are only slightly lower than the amount that is being withdrawn, hence the fund size decreases very gradually. However, this leaves a large proportion of the fund untouched. The 10th percentile income drops rapidly at the start but this fall slows at later ages, while the 90th percentile rises to almost 150% of the start value.

- In the 5.59% case (see Figure 2), we find that the median steadily reduces as the rate of withdrawal significantly outweighs the rate of return after management fees. The median case returns more than an annuity until the age of 73 beyond which it drops to half the annuity rate at age 100. The 10th percentile drops to half the annuity rate at age 76 and converges toward 1p per £1 after age 100. The 90th percentile rises to a peak at 7.35p per £1 invested at age 78 and slowly drops back to near 5.5p per £1 at age 100.

Figure 1: Fixed Percentage - 4.66%

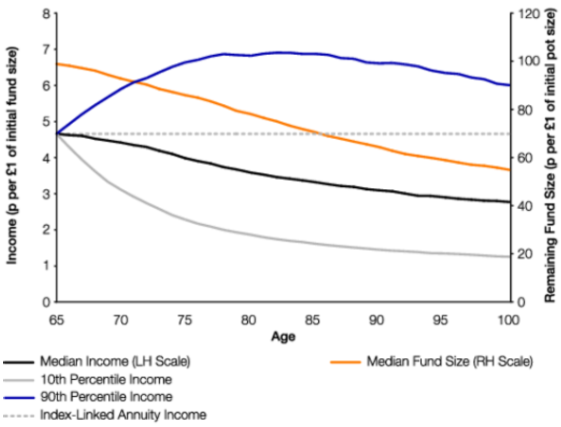
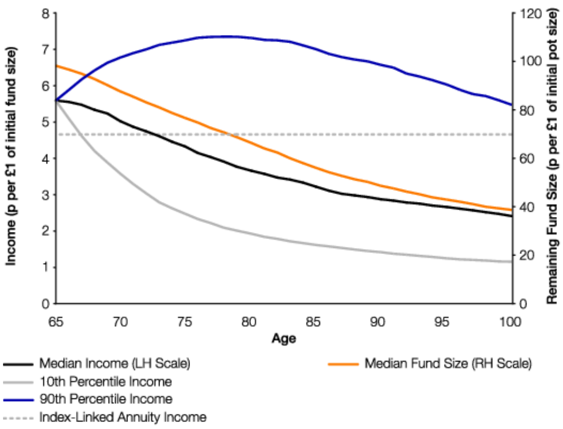


Figure 2: Fixed Percentage - 5.59%



The great advantage of this strategy is its simplicity. Every year, a set percentage of what is left is removed. However, the balance between fund size and withdrawal rate is not good from the point of view of gradual consumption of pension assets through retirement. Where the withdrawal rate is similar to the expected rate of return on the investment, the pension fund depletes very slowly. If the bequest motive is incredibly strong, perhaps this may be an acceptable strategy for an individual, but probably not from a public policy perspective

Variable Percentage

Rather than a fixed percentage, it is possible to construct an approach whereby the percentage actually varies over time depending on the retiree's age:

$$B_t = \omega_t V_t$$

Where B_t is the benefit received in year t after retirement, ω_t is the percentage in year t and V_t is the size of the pension fund in year t.

We present four different variations of ω with respect to t:

Linear Increase:

This approach tries to take account of the falling fund size, and thus falling income, by increasing the percentage that can be withdrawn as the pension holder gets older (see Figure 3). Starting from a percentage withdrawal of 5.13%, which is 110% of the index-linked annuity rate for a 65 year old man, the withdrawal rate is linearly increased to a maximum of 23.13% for a 110 year old man. Thus:

$$\omega_t = 0.0513 + 0.04 * t$$

In this strategy, there is a delicate interaction between the withdrawal rate, the fund size and the rate of investment return:

- We find that for the median case the income rises to a peak of 6.4p per £1 at age 71 and then gently falls reaching annuity rate at age 82 and continuing to fall beyond.
- The 90th percentile case rises more rapidly and peaks a little later hitting 10.3p per £1 at age 77 and not getting back down to annuity rates until age 91.
- The 10th percentile gently slopes from the annuity rate downwards. For the majority of its life the fund size decreases almost linearly tailing off towards very old age.

Again, this strategy is fairly simple – in year t, x% is withdrawn. However, the fact that the income peaks some years into retirement could be a marked disadvantage, as is the fact that in the 10th percentile the income stream does not hit an equilibrium. Nonetheless, the median case provides an income stream above that of an annuity for the first 15 years – the years in which consumption would be expected to be at its highest and the chance of being alive highest.

Figure 3: Variable Percentage – Linearly Increasing

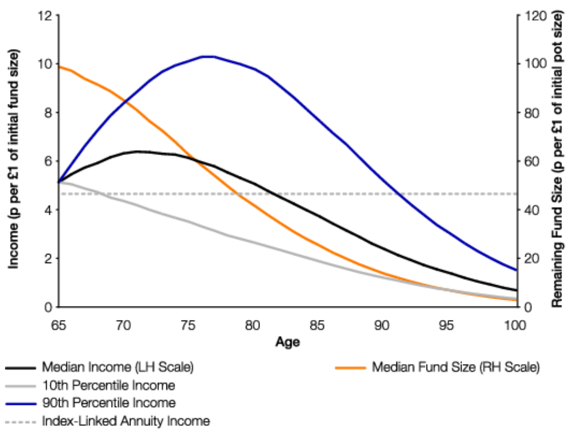
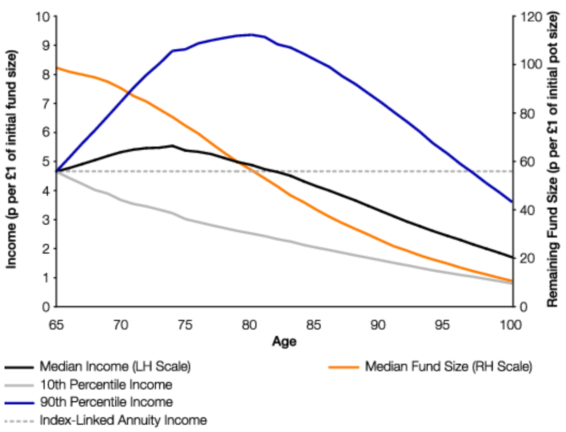


Figure 4: Variable Percentage – Equivalent Annuity



Equivalent Annuity:

For the years between 65 and 75, ω_t is equal to the current index-linked annuity rate for a man of that age. Beyond 75, where an annuity rate is not quoted, we extrapolate the annuity rate to make a prediction of possible annuity rates for a 76 year old, 77 year old etc. (see Figure 4):

- Again, we see that the median case rises above the value of an annuity reaching a peak of 5.5p per £1 and falling back to the amount that would be received from an annuity at age 82.
- The 90th percentile increases more rapidly to a peak of 9.4p per £1 at age 80 and remains above the annuity rate until age 97.
- The 10th percentile, however, steadily declines almost linearly from the annuity rate at age 65 to less than half at age 82 and beyond.

The fund size falls steadily throughout the life of the pension holder.

Canadian RRIF:

The Canadian system is technically a variable percentage withdrawal strategy. It uses the Canadian withdrawal rules for their Registered Retirement Income Fund (RRIF) pension scheme that determine the minimum value that can be withdrawn each year. This ensures that pension savings, and the tax relief that they attract, are actually used for the purpose of providing a retirement income.

This is a real life example and one that is designed such that the income is comparable to that of a level annuity. Consequently, the percentage withdrawals are all in nominal terms and the income stream is optimised to be flat in nominal terms. This means that in order to judge this method on the grounds upon which it was created, we had to run the simulation in nominal terms. To do this we assumed an inflation rate of 3% and increased the mean real returns that are inputted into the Monte Carlo simulation accordingly.[14]

The RRIF scheme has the following withdrawal rules. Between the ages of 71 and 94, the Canadian Government sets the minimum percentage of the remaining pension fund that can be withdrawn. This increases from 7.38% to 20% at age 94. For pension holders over 94 it remains at 20% of the remaining pot. There are no maximum withdrawal limits at any point.

For those pension holders that are under 70 and want to withdraw from their pot, they have to take a minimum of 1/(90-A) where A is the age. This results in a large jump from age 70 to 71 (5% to 7.38%). To avoid this jump, and completely in compliance with Canadian rules, we smooth the withdrawals and take larger percentages pre-age 70 extrapolated back from the trend between 71 and 94.

[14] Our calculation comes from the implied inflation rate due to the difference in yields on index-linked treasury gilts and nominal treasury gilts. According to the Bank of England spot curve the implied inflation figure for a 20 year maturity is approximately 3.4%. Recognising that index-linked gilt yields are artificially low due to the supply constraints we use an interest rate a little lower than the 3.4%, hence we chose 3.0%.

Figure 5: Canadian RRIF in Nominal Terms

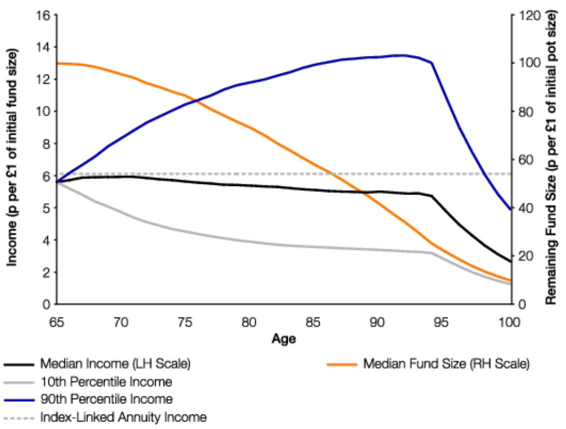


Figure 6: Canadian RRIF in Real Terms

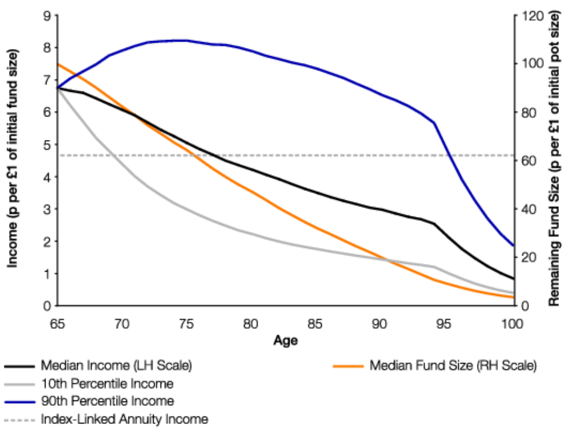


Figure 5 shows the income stream in nominal terms and then Figure 6 shows what happens when this income stream is deflated by inflation of 3% to make it comparable to the index-linked annuity. With the exception of the pre-70 phase we have modelled withdrawing the minimum percentages.

Considering the income stream in nominal terms:

- This results in the median case being a fairly steady withdrawal that very slightly decreases. The income level lies just below that of a level annuity, which as of 3rd October 2007 was 7.2%. After the age of 94, there is a fairly rapid drop in the income as the withdrawal percentage is fixed at 20%, but the fund is decreasing in size.
- The 90th percentile has the same features but rises to approximately twice the value of an annuity (14.1p per £1) at age 94. It drops back to an annuity rate aged 99.
- For the 10th percentile case, the income drops to equilibrium at about 3p per £1 or 42% of the value of an annuity.
- The size of the median fund stays pretty level until age 70 when it starts to decrease at a steady rate until it is 10% of the original size at age 100.

Considering this strategy in real terms:

- The median profile starts considerably higher than the value of the index-linked annuity. It remains above the annuity until age 78. Inflation has a major effect, fairly rapidly eroding the real value of the income stream such that it is worth 2.5p per £1 by age 95. After the age of 95, the income drops more rapidly.
- The 90th percentile fares a lot better. It hits a peak of 8.25p at age 76 and remains above the value of an annuity until age 96.
- The 10th percentile drops very rapidly as inflation wears down the value of the income stream. It crosses the annuity rate at age 70 and half the annuity rate at age 82. After that it drops rather less rapidly.

While this falling income stream matches the fact that the expected consumption profile decreases with age it suffers from the same problem as a level annuity in that it is very vulnerable in the face of unexpected inflation.

Exponential Increase:

We exponentially increased from 5.13%, which is 110% of the current index-linked annuity rate of 4.66% for a 65 year old man, to almost 100% for a 110 year old man following the equation below:

$$\omega_t = 0.0513 + 0.008 * (e^{0.11*(t-65)} - 1)$$

This is an attempt to marry the falling fund size with an increasing percentage withdrawal that ends up resulting in a relatively steady withdrawal rate (see Figure 7):

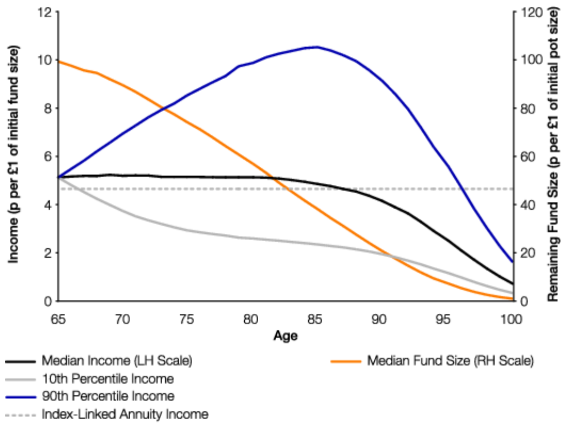
In the median case, we find that the income exceeds an annuity rate until the age of 88 when it starts to fall more rapidly. For the first 23 years, the income stays more or less level at about 110% of the equivalent annuity rate. It starts to drop below the annuity rate after age 87.

In the 90th percentile case, the income grows more slowly than in the linearly increasing percentage approach. This allows more potential for growth within the fund and it hits a maximum of 10.5p per £1, over twice the annuity rate, at age 85. It then falls rapidly as the fund decreases in size crossing the annuity rate at age 97.

In the 10th percentile case, the income slowly drops crossing half the annuity rate at age 86. It drops very slowly, particularly in the period 75-90.

The median fund is used up almost linearly for the majority of the pension holder's life coming close to, but not actually, running out at age 100.

Figure 7: Variable Percentage – Exponentially Increasing



This method has a few advantages. The exponentially increasing percentage combined with the decreasing pot leaves the income profile fairly level for the median case until later in retirement. In the 10th percentile (i.e. 90% of the time an individual will do better), the income doesn't drop below half of an annuity until around age 86, whereas the peak of the 90th percentile is well over twice the annuity rate. The income also tends to drop towards the far end of retirement (>85 years old) when it is expected that consumption would also drop.

1/E(t)

With this approach, the amount withdrawn is dependent on the pension holder's life expectancy at the given year E(t). These values we use of E(t) are from the CMI "92" series Mortality Tables[15] combined with the projection for the predicted increase in life expectancy.[16] We calculate E(t) as the sum of the chance that a person aged t will be alive each year between t and 110.

$$E(t) = \sum_{x=t}^{110} P_t^x$$

Where P_t^x = the probability that a person aged t will live to age x. We do this for both the annuitant population and the population as a whole.

The benefit that is received each year is then calculated as:

$$B_t = \frac{V_t}{E(t)}$$

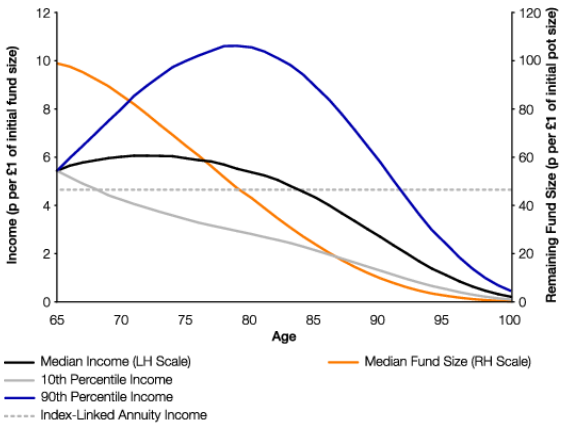
There are 2 sets of life expectancies we have used to present this data. The first is the population as a whole, where for all percentiles the initial withdrawals are considerably lower than the annuity rate. Over time, the fund is able to grow as the withdrawal rate is lower than the investment return (see Figure 8):

- In the median case, we find that the withdrawal amount starts off greater than that of an annuity, and it increases, reaching a peak of 6.1p per £1 at age 71 before dropping back below the annuity rate at age 84.

- The 90th percentile case peaks rather higher and rather later at 10.6p per £1, almost 2.5 times the annuity rate, age 79. It then rapidly falls, crossing the annuity rate at age 92.

- The 10th percentile starts at a rate higher than the annuity. It crosses the annuity rate aged at age 69 and hits half the annuity rate at age 84.

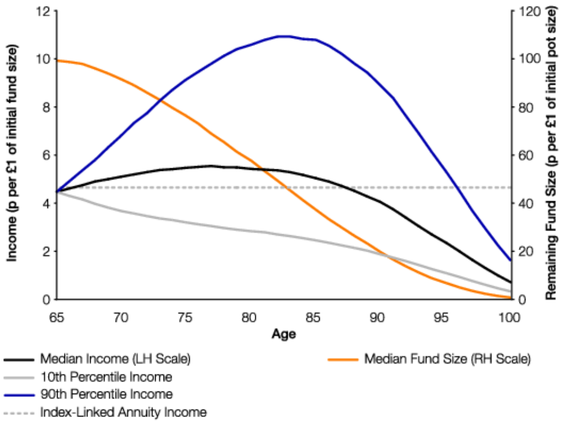
Figure 8: 1/E(T) – Based on Entire Population Mortality



[15] Continuous Mortality Investigation Reports No 17 (CMIR17).

[16] Continuous Mortality Investigation - Mortality Improvement Model For Use With the "92" Series of Mortality Tables.

Figure 9: 1/E(T) – Based on Annuitant Mortality



The second set of life expectancies is a subset of this population who are annuitants. The annuitant population has lighter mortality than the population as a whole, (i.e. they live longer), and so we find that their life expectancy at each age is higher. This means that the percentage withdrawn each year will be smaller as it is inversely proportional to the life expectancy. The initial withdrawals are, therefore, lower than those of the entire population. This results in additional growth potential for the pension fund, which depletes very slowly in the first few years despite the fact that we are withdrawing from it (see Figure 9).

This has a knock-on effect on the income in later years:

- In the median case, we find that the withdrawal amount is less than that of an annuity until age 67, when it increases, reaching a peak of 5.6p per £1 at age 77 before dropping back below the annuity rate at age 88.
- The 90th percentile case peaks rather higher and rather later at 10.9p per £1, almost 2.5 times the annuity rate, at age 83. It then rapidly falls, crossing the annuity rate at age 96.
- The 10th percentile drops fairly slowly hitting half the annuity rate at age 86.

The inherent problem with this method is the peak of income arriving well into retirement. Looking at general consumption patterns, we see that spending tends to gradually fall during retirement and a profile of income which peaks at around age 77 for the annuitant population is perhaps not the best use of a pensioner's savings. Another problem is that as life expectancy changes, in theory, the annual percentages need to change, which further complicates the system for the pensioner.

3. A Quantitative Discussion of Simulation Results

Money's Worth

To present a more concrete quantitative case, we use the concept of Money's Worth. Money's Worth has been used to determine whether or not the price of an annuity is actuarially fair. It essentially compares the expected discounted amount payable to a pensioner to the amount that they paid for their annuity. A Money's Worth value of one indicates that an annuity is 100% actuarially fair: i.e. after factoring for bond returns, an individual can expect to get out exactly what they put in. Of course, in reality, we don't expect the value to be quite one due to the costs borne by the pension provider in administration, management etc. along with a degree of profit. It is generally argued that a fair value for Money's Worth is a little less than one – how much less than one is considered fair is a matter of debate.[17] It has been noted that the Money's Worth value of annuities is consistently in the range 0.85-1.05.[18]

There is some debate about how to extend the Money's Worth calculation beyond its conventional use – i.e. a way of measuring the actuarial fairness of annuities. It has been suggested in part of the literature that the discount rate should reflect consumers' risk preferences and that in effect, £1 of bonds should have the same Money's Worth as £1 of stocks assuming an appropriate (i.e. risk adjusted) discount rate has been used.[19]

In the approach that we use, the Money's Worth concept is intended only to provide a way of comparing the potential income flows from different drawdown strategies to those provided by an annuity. It is not a 'value for money' statement, and we have not modelled a utility function which takes account within the Money's Worth value of the different risks that an individual is exposed to in the drawdown product. To do so appears highly problematic given the range of risk preferences that are likely to exist in different products and/or combination of products.

A Money's Worth value of 1.30 would effectively show that a given drawdown strategy could generate £1.30, compared to £1 invested at the risk free rate. However, to reiterate, the same Money's Worth ratio cannot be said to have the same direct value or utility to the individual in the drawdown approach as in the annuity, where the Money's Worth is based on certainty rather than expectation based on a distribution of possible outcomes. However, by presenting the drawdown results across different percentiles, we do demonstrate the range of potential outcomes.

[17] For more detailed discussion see Cannon and Tonks (2004), Martin and Fitzgerald (2006), Murthi, Orszag and Orszag (1999).

[18] HM Treasury (2006), p.11.

[19] For further discussion see Geanakoplos (1998) and James and Song (2001).

The Money's Worth value is calculated by summing the product of all the expected payments with the chance that an individual will be alive when the payment is due to be made. In the case of the annuity, this is the guaranteed annuity rate. For the simulated income drawdown strategies, this is the payment received in the given year. These payments are discounted at a rate that represents the long term index-linked bond return. We use 1.27% which was the yield on 1¼% Index-linked Treasury Gilt 2027 on 3rd October 2007.[20] The total is then divided by the amount paid for the premium. So Money's Worth is calculated using the following formula:

$$MW = \frac{\sum_{t=65}^{t=110} \frac{B_t}{(1+r)^{t-65}} P_{65}^t}{Y}$$

Where B_t = benefit paid out at year t, r = risk-free bond rate, P_{65}^t = probability of a person aged 65 living until aged t and Y = initial premium paid. P_{65}^t is calculated by multiplying together the chances of not dying for all the years between 65 and t:

$$P_{65}^t = \prod_{x=65}^{t} (1 - q(x))$$

Where q(x) is the mortality rate aged x.

Predictions of mortality rates, q_t, are based upon the Continuous Mortality Investigation "92" series of tables. These are then combined with the CMI "Mortality Improvement Model" which adjusts these mortality figures and projects them forward to account for expected improvements in mortality rates. We project for a man who retires aged 65 in 2007 (i.e. experiences the mortality of a male of 66 in 2008 etc.). The projection uses the following factor to project forward from the base year of 1992.

$$RF(x,t) = \alpha(x) + [1-\alpha(x)][1-f(x)]^{\frac{t}{20}}$$
$$\alpha(x) = 1 + (1-c)\frac{(x-110)}{50}$$
$$f(x) = \frac{(110-x)p + (x-60)q}{50}$$

Where c=0.13, p=0.55, q=0.29, t = the number of years projected forward beyond 1992 and x=age.

We consider Money's Worth based upon two different mortality tables. The first gives the mortality rates for men throughout the whole population. The second is a subset of the first and includes only the annuitant population. The annuitant population tends to have lighter mortality than the population as a whole. The result of this is higher values of Money's Worth when calculated with the annuitant tables, as the pensioner is more likely to be alive to receive the payment than in the whole population case. We present both sets of figures.

In the presentation of the Money's Worth for income drawdown strategies, we consider the median value of the 10,000 simulations for each withdrawal strategy. We include a table that also gives the Money's Worth values for the 10th and 90th percentiles as Table 5 shows.

[20] Source: UK Debt Management Office.

Table 5: Money's Worth Across Different Withdrawal Strategies

Withdrawal Strategy	Money's Worth for Entire Population	Money's Worth for Annuitants
	Median	Median
Annuity	-	0.93
Fixed Value 4.66p per £1	0.78	0.91
Fixed Value 5.59p per £1	0.87	0.98
Fixed Percentage 4.66%	0.67	0.78
Fixed Percentage 5.59%	0.74	0.84
Linearly Increasing Percentages	0.88	0.99
Exponentially Increasing Percentages	0.84	0.97
Equivalent Annuity Percentages	0.82	0.93
Canadian RRIF	0.86	0.96
1/E(t) – All	0.90	-
1/E(t) - Annuitants	-	0.97

The first thing that we can see from Table 5 is that annuities are fairly priced. The current price for an index-linked annuity of 4.66% (for a 65 year old non-smoking male as of 3rd October 2007) provides a total expected income of 93% of the money used to buy the annuity after discounting for bond returns. Index-linked annuities tend to have a lower Money's Worth value than an equivalent level annuity. When we apply the same calculation to the equivalent level annuity discounted by a nominal bond rate we find that the annuity has Money's Worth of 99%.

There may be a number of reasons why index-linked annuities have lower Money's Worth values: the provision of an inflation-linked income incurs higher costs, index-linked bond yields may be suppressed due to the limited supply of index-linked bonds that are issued and the lack of many other instruments with which to cover inflation risk and group selection effects whereby those who buy the index-linked product have a longer life expectancy as they expect to need inflation protection.[21]

However, if the annuitant population were to broaden to be more representative of the population as a whole, the annuity rate would need to increase to 5.50% to provide the same Money's Worth value at current bond rates. In every strategy simulated the Money's Worth figure is higher for the annuitant population than the entire population. This is what we would expect given that annuitant mortality is lighter than that of the whole population (i.e. they are less likely to die each year). There is a greater chance of the annuitant living to receive the payment each year than the average member of the general population.

[21] See Cannon & Tonks (2006) and Finkelstein and Poterba (2002).

With respect to the different drawdown solutions, there are widely differing Money's Worth outcomes, as would be expected:

- Fixed Value. If an individual self-annuitises at the annuity rate, the Money's Worth is inevitably less than that of an annuity as there is a chance of running out of money that is not present with an annuity. If the amount withdrawn each year is increased to 5.59p per £1, the chances of running out of money increases. Despite this increased chance of running out of money, the Money's Worth value increases as the years in which an individual would receive nothing are the years in which they are least likely to be alive. In this case, the Money's Worth value exceeds that of an annuity.

- Fixed Percentage. The fixed percentage strategy fares badly. For the 5.59 fixed percentage, this is because the expected return, after management fees, on the portfolio is less than 5.59%. This means that the fund decreases in value year upon year, which in turn means that the income decreases in monetary terms and quickly drops below that of an annuity which is a fixed value for all years. The result is a Money's Worth of 0.84. For the 4.66% fixed percentage, the income stream also drops but to a lesser degree than the 5.59% case as the expected return, after management fees, is closer to the withdrawal percentage. However, because it starts at a lower value, the Money's Worth value is even lower at 0.78.

- Variable Percentage. With respect to percentage withdrawals that vary with age (to take account of reducing fund size), we see that it is possible to construct a number of different strategies that lead to a Money's Worth value greater than that of an annuity. In fact, for a linearly increasing percentage, the median Money's Worth is slightly shy of unity: i.e. an individual can expect to get back almost all they started with in income payments. The Canadian RRIF percentage is the Money's Worth of the deflated income stream so that it is suitable for comparison. In all tables, the Money's Worth and bequest values for the Canadian RRIF are presented in real terms.

- 1/E(t). We see that the 1/E(t) strategies for withdrawal have Money's Worth values that surpass that of an annuity when compared to the same population.

However, there are two elements to bear in mind when considering these numbers:

- The figures we quote are median values – to consider the mean, and thus expected value, add approximately 0.05 to all the Money's Worth values except that of the annuity. This further increases the value of the non-annuity withdrawal strategies.

- The Money's Worth value only considers the income payments that a pensioner receives; it neglects the fact that with all the alternative withdrawal strategies there is considerable scope for bequests. We calculate the total fund value (including potential for bequest) in the section on overall return.

Table 6: Money's Worth Across Different Withdrawal Strategies – 10th and 90th percentiles

Withdrawal Strategy	Money's Worth for Entire Population		Money's Worth for Annuitant Population	
	10th Percentile	90th Percentile	10th Percentile	90th Percentile
Annuity	-	-	-	-
Fixed Value 4.66p per £1	0.62	0.79	0.66	0.93
Fixed Value 5.59p per £1	0.66	0.94	0.69	1.11
Fixed Percentage 4.66%	0.45	1.03	0.50	1.23
Fixed Percentage 5.59%	0.50	1.14	0.56	1.34
Linearly Increasing Percentages	0.59	1.35	0.65	1.55
Exponentially Increasing Percentages	0.55	1.32	0.62	1.57
Equivalent Annuity Percentages	0.53	1.26	0.60	1.49
Canadian RRIF	0.60	1.27	0.65	1.47
1/E(t) – All	0.60	1.38	-	-
1/E(t) - Annuitants	-	-	0.62	1.60

Compared to the median across all strategies, we find that there is approximately a 30% drop in the Money's Worth values for the 10th percentile case but this is balanced by a 44% increase in Money's Worth in the 90th percentile case. If we ignore the fixed benefit strategies where the income stream does not depend on investment return we find that the upside benefit increases to 54% compared to a drop of 32% in the 10th percentile case.

Sensitivity Analysis

Management Charges

We have assumed an annual management charge of 1%, given that the models are based upon what would in effect be passively managed index-tracking products. However, it is clearly necessary to consider what Money's Worth would look like in a world with a higher annual management charge (AMC).

Table 7: Effect of Higher Management Charges on Money's Worth

Withdrawal Strategy	Money's Worth for Annuitants – AMC 1%	Money's Worth for Annuitants – AMC 1.5%	Money's Worth for Annuitants - AMC 2%
	Median	Median	Median
Annuity	0.93	0.93	0.93
Fixed Value 4.66p per £1	0.91	0.88	0.85
Fixed Value 5.59p per £1	0.98	0.94	0.91
Fixed Percentage 4.66%	0.78	0.74	0.70
Fixed Percentage 5.59%	0.84	0.81	0.76
Linearly Increasing Percentages	0.99	0.94	0.91
Exponentially Increasing Percentages	0.97	0.92	0.87
Equivalent Annuity Percentages	0.93	0.89	0.85
Canadian RRIF	0.96	0.92	0.89
1/E(t) - Annuitants	0.97	0.92	0.88

Increasing the AMC inevitably decreases the income stream and thus the Money's Worth value. For a fixed value this manifests itself in the year at which the money runs out. In this case either it runs out earlier or the final year's payment is smaller than it would have been. In the other cases the annual payout is slightly decreased.

From the table above we can see that every 0.5% increase in the AMC has the average effect of decreasing the Money's Worth by about 0.04. This is not entirely uniformly spread. The withdrawal strategies that have the largest withdrawals later in life (i.e. 1/E(T)) suffer more seriously as there is more time for the accumulation of the loss of return on the charges that are removed from the fund. As this loss accumulates, the Money's Worth falls.

While certain withdrawal strategies compare favourably to the Money's Worth for the annuity at a 1% AMC, this is clearly not the case with a 2% AMC.

However, it is important to note once again that to compare the true fund value, one would have to include the bequest potential.

Asset Allocation

An optimal asset allocation balances the desire for a high return with the chance of, and magnitude of, loss in the event of poorer than expected returns. Looking across the income drawdown strategies that we have modelled, an increase in equity exposure raises the Money's Worth value for the median profile but also increases the spread of possible outcomes, i.e. the range between the 10th and 90th percentiles. An increase in the share of gilts in the portfolio has the opposite effect; the Money's Worth of the median profile decreases, but with it the range of outcomes decreases, giving the portfolio more certainty about the outcome. This is illustrated in Table 8 below.

Table 8: Effect of Change of Asset Allocation

Withdrawal Strategy	70:10:20 Money's Worth (1)	60:20:20 Money's Worth	50:30:20 Money's Worth	70:10:20 Top-Bottom Spread (2)	60:20:20 Top-Bottom Spread	50:30:20 Top-Bottom Spread
Fixed Value 4.66p per £1	0.92	0.91	0.88	0.27	0.26	0.27
Fixed Value 5.59p per £1	1.00	0.98	0.95	0.42	0.42	0.42
Fixed Percentage 4.66%	0.81	0.78	0.74	0.85	0.73	0.64
Fixed Percentage 5.59%	0.87	0.84	0.81	0.86	0.78	0.68
Linearly Increasing Percentages	1.04	0.99	0.95	1.02	0.90	0.80
Exponentially Increasing Percentages	1.02	0.97	0.92	1.09	0.96	0.83
Equivalent Annuity Percentages	0.98	0.93	0.90	1.06	0.890	0.789
Canadian RRIF	1.01	0.96	0.93	0.94	0.82	0.75
1/E(t) - Annuitants	1.01	0.97	0.93	1.13	0.98	0.86
Average (ignoring Fixed Value)	0.96	0.92	0.88	0.99	0.87	0.76

1. The ratios expressed are Equities:Gilts:T-bills. Hence 70:10:20 means that portfolio is made up of 70% Equities, 10% Gilts and 20% T-bills.
2. "Top-Bottom Spread" means the difference between the Money's Worth values of the 90th and the 10th percentiles.

We elected for a 60:20:20 asset mix in our simulations presented in this paper as this balances the desire for a reasonably high return with the desire to increase the certainty of the outcome by restricting the equity exposure. With this asset allocation, the downside risk, measured as the difference between the Money's Worth of the median and 10th percentiles, is only 60% of the possible upside benefit, measured as the difference between the Money's Worth of the 90th percentile and the median. Furthermore, the average of the median Money's Worth results is comparable to that of an annuity (0.93, see page 8).

Bequest Potential

An advantage of income drawdown is the bequest potential. In theory, the remaining fund would transfer directly to the estate of the deceased. We expect that this would be taxed at a suitable rate to allow for the fact that a significant proportion of pension wealth exists as a result of tax incentives that are in place during the accumulation phase to encourage saving. However the tax element is treated, there remains the potential for a bequest. The expected bequest is calculated as the sum of the size of fund at year t multiplied by the chance that the pension holder will live from 65 and die at year t, and discounted by the risk free rate.

$$E(B) = \sum_{t=65}^{110} \frac{V_t . q(t) . \prod_{x=65}^{t-1} (1 - q(x))}{(1+r)^{t-65}}$$

Where E(B) = Expected Bequest, q= mortality rate, V_t=pot size at year t, r=risk-free rate (1.27% as in Money's Worth calculation).

In order to allow the bequest value to be compatible with the Money's Worth value, we discount the bequests by the risk free rate. A bequest value of 0.45 tells us that we can expect a bequest of 45p per £1 of initial fund value compared to if that £1 was invested at the risk free rate.

Our findings are presented in Table 9.

Table 9: Bequest Estimates Across Different Strategies and Populations

Withdrawal Strategy	Expected Bequest for Entire Population	Expected Bequest for Annuitant Population
	Median	Median
Annuity	0	0
Fixed Value 4.66p per £1	0.45	0.33
Fixed Value 5.59p per £1	0.29	0.19
Fixed Percentage 4.66%	0.60	0.53
Fixed Percentage 5.59%	0.51	0.44
Linearly Increasing Percentages	0.32	0.23
Exponentially Increasing Percentages	0.39	0.28
Equivalent Annuity Percentages	0.41	0.31
Canadian RRIF	0.35	0.25
1/E(t) – All	0.31	-
1/E(t) – Annuitants	-	0.28

It is not surprising to see that the expected bequest for the entire population is always higher than that of the annuitant population. This is due to the fact that the annuitants tend to live longer than average and so will have used more of their savings as income when they do come to die. The difference that this lighter mortality rate makes is around 10% of the initial pension fund. Table 11 includes the 10th and 90th percentile values.

Overall Return from Pension Fund

This section presents the total accumulated fund value (i.e. income stream plus bequest potential). We calculate this by adding the Money's Worth value for the income stream to the bequest value. We then calculate the possible total return for the pension holder and his/her heirs by considering a suitable tax rate applied to the bequest. To calculate this we assume a hypothetical tax rate applied to bequests of 55% of the remaining fund upon death of the pension holder. We choose 55% as the Government states in "The Annuities Market" of December 2006 that "more than half of this fund (that of a higher rate tax payer) could consist of accumulated tax relief". This is supported by our own calculations based upon the contributions of a higher rate taxpayer.

In every case – even after 55% tax applied to the bequest – the median total received by the pension holder and his/her heirs considerably exceeds that of an annuity. In fact, the median return is between 9% and 17% better than an annuity.

We find that in the median case the annuitant and general population values become closer.

This is essentially due to the fact that the whole population has a shorter life expectancy so receives less during their life but consequently leaves a larger bequest. The annuitant group still has a higher value as it lives longer so the pot has more time to benefit from investment returns. The results for the percentiles are found in Table 14 and Table 15.

Table 10: Overall Return on Fund and Estimated Return for Pension Holder and Heirs

Withdrawal Strategy	Total MW and Bequest - Entire Population	MW + Bequest Taxed at 55% – Entire Population	Total MW and Bequest - Annuitants	MW + Bequest Taxed at 55% – Annuitants
	Median	Median	Median	Median
Annuity	-	-	0.93	0.93
Fixed Value 4.66p per £1	1.23	0.98	1.23	1.05
Fixed Value 5.59p per £1	1.17	1.01	1.17	1.06
Fixed Percentage 4.66%	1.27	0.94	1.31	1.02
Fixed Percentage 5.59%	1.24	0.96	1.27	1.03
Linearly Increasing Percentages	1.20	1.03	1.22	1.09
Exponentially Increasing Percentages	1.23	1.01	1.25	1.09
Equivalent Annuity Percentages	1.23	1.00	1.25	1.07
Canadian RRIF	1.20	1.01	1.21	1.08
1/E(t) – All	1.21	1.04	-	-
1/E(t) – Annuitants	-	-	1.25	1.10

Conclusion

Income drawdown offers both exposure to the potential returns of equity investment and the possibility of a bequest. When these factors are combined (even after considering a 55% tax on bequests), the median amount, in Money's Worth terms, returned to the pension holder and his/her heirs exceeds that of the annuity in all strategies that we have considered.

We have also shown that it is possible to have a system with a relatively stable income, which hits an equilibrium that depends upon the investment returns, but whose median pathway provides a higher income than that of an annuity. At the same time, appropriate design can deliver higher withdrawals in the early years of retirement, gradually tailing off as the pension holder gets older. While some drawdown strategies appear more viable than others, all have their own combination of advantages and disadvantages (presented in summary form in Table 11).

There are of course inherent risks within income drawdown strategies – namely that the pension holder will run out of money or that the income will decrease to an unacceptable value. The first risk can be prevented by a strategy that takes a percentage of the remaining fund and not a fixed value. This ensures that the fund is never exhausted. The second risk is unavoidable if the possibility of higher investment returns is desired. It can certainly be minimised with an appropriate investment strategy but can never be eliminated altogether.

We observed, however, that although this risk is very real, over the timescales we considered it is not high. For example, the 10^{th} percentile for the exponential increasing percentage withdrawal case does not drop below half of the annuity income until age 86. At the same time, the 90^{th} percentile almost hits 200% of the annuity rate. In fact, in all strategies looked at we found that the downside risk is considerably smaller than the potential upside gain and this fits well with a concave utility function of wealth.

As we discuss in more detail in the accompanying discussion paper, these research findings should not be read as suggesting that annuities do not have a significant role to play in the retirement product market. However, the Government's emphasis on annuity based products is effectively preventing individuals from exercising a wider choice that could result in considerably improved retirement income.

APPENDIX

Table 11: Summary of Potential Advantages and Disadvantages of each Withdrawal Strategy

	Income Stream (median)	Pot Size (median)	Main Potential Advantages	Main Potential Disadvantages
Fixed Value	Fixed at either annuity rate or 120% annuity rate.	Steadily drops to zero – age at which it runs out depends upon rate of withdrawal.	Simple design. Can receive larger income than annuity. Income fixed in nominal terms.	Income does not benefit from investment return. Decreasing income in real terms. May run out if individual lives too long or suffers poor investment returns.
Fixed Percentage	Gently decreasing income in nominal terms.	Pot drops slowly for 4.66%, drops to about 40% of the initial size for a withdrawal rate of 5.59%.	Simple design. Large bequest potential.	Decreasing income in real terms. Leaves capital relatively untouched. Income variable.
Variable Percentage				
1. Linear increase	Rises to peak at age 71 then gradually decreases.	Almost linearly decreases heading towards 0 past age 100.	Simple design. Median Money's Worth for income better than annuity. 90th percentile peak arrives about age 76.	Income falls below that of an annuity age 82. 10th percentile continually falls. Income variable.
2. Exponential increase	Flat income until age 85. Gently decreases afterwards.	Drops almost linearly towards 0 past age 100.	Median Money's Worth for income better than annuity. 10th percentile decreases slowly. Income better than annuity until age 88.	Complex design. Income variable.
3 Equivalent Annuity	Rises to a peak at age 74 and then drops gently.	Decreases gently at first and then nearly linearly thereafter.	Median Money's Worth for income better than annuity. Pays out more than annuity until mid 80's and does not drop too rapidly after.	Complex design. Income variable. Income peaks at age 74 and drops in real terms from then on.
4. Canadian RRIF	In nominal terms very gradually drops towards age 95 and then rapidly after. In real terms, drops rapidly.	Flat for first 4 years and then drops gradually.	Median Money's Worth for income better than annuity. Median income stream steady in nominal terms. Median income better than index-linked annuity until age 78. 10th percentile stabilises in nominal terms.	Complex design. Income variable. Income falls sharply in real terms.
1/E(t)	Rises to a late peak – late 70's to early 80's – and then drops off after that.	Drops slowly in early years and then drops off quickly in later years.	Median income better than annuity between age 65 and 84 based on entire population mortality.	Complex design. Income variable. Income peaks late in retirement. Income too low in early years.

Table 12: Money's Worth across Different Withdrawal Strategies (including percentiles)

Withdrawal Strategy	Money's Worth for Entire Population			Money's Worth for Annuitants		
	10th Percentile	Median	90th Percentile	10th Percentile	Median	90th Percentile
Annuity	-	-	-	-	0.93	-
Fixed Value 4.46p per £1	0.62	0.78	0.79	0.66	0.91	0.93
Fixed Value 5.59p per £1	0.66	0.87	0.94	0.69	0.98	1.11
Fixed Percentage 4.46%	0.45	0.67	1.03	0.50	0.78	1.23
Fixed Percentage 5.59%	0.50	0.74	1.14	0.56	0.84	1.34
Linearly Increasing Percentages	0.59	0.88	1.35	0.65	0.99	1.55
Exponentially Increasing Percentages	0.55	0.84	1.32	0.62	0.97	1.57
Equivalent Annuity Percentages	0.53	0.82	1.26	0.60	0.93	1.49
Canadian RRIF	0.60	0.86	1.27	0.65	0.96	1.47
1/E(t) – All	0.60	0.90	1.38	-	-	-
1/E(t) - Annuitants	-	-	-	0.62	0.97	1.60

Table 13: Expected Bequests across Different Withdrawal Strategies (including percentiles)

Withdrawal Strategy	Expected Bequest for Entire Population			Expected Bequest for Annuitant Population		
	10th Percentile	Median	90th Percentile	10th Percentile	Median	90th Percentile
Annuity	0	0	0	0	0	0
Fixed Value 4.66p per £1	0.13	0.45	1.45	0.08	0.33	1.50
Fixed Value 5.59p per £1	0.10	0.29	1.18	0.06	0.19	1.14
Fixed Percentage 4.66%	0.33	0.60	1.12	0.27	0.53	1.08
Fixed Percentage 5.59%	0.28	0.51	0.96	0.22	0.44	0.90
Linearly Increasing Percentages	0.19	0.32	0.54	0.13	0.23	0.40
Exponentially Increasing Percentages	0.23	0.39	0.67	0.16	0.28	0.51
Equivalent Annuity Percentages	0.24	0.41	0.71	0.17	0.31	0.56
Canadian RRIF	0.210	0.35	0.58	0.15	0.25	0.45
1/E(t) – All	0.19	0.31	0.52	-	-	-
1/E(t) - Annuitants	-	-	-	0.16	0.28	0.51

Table 14: Overall Return on Fund and Estimated Return for Pension Holder and Heirs based upon Entire Population Mortality Tables.

Withdrawal Strategy	Total MW and Bequest - Entire Population			MW + Bequest Taxed at 55% – Entire Population		
	10th Percentile	Median	90th Percentile	10th Percentile	Median	90th Percentile
Annuity	-	-	-	-	-	-
Fixed Value 4.66p per £1	0.75	1.23	2.24	0.68	0.98	1.44
Fixed Value 5.59p per £1	0.76	1.17	2.12	0.70	1.01	1.47
Fixed Percentage 4.66%	0.77	1.27	2.15	0.60	0.94	1.54
Fixed Percentage 5.59%	0.78	1.24	2.10	0.63	0.96	1.57
Linearly Increasing Percentages	0.78	1.20	1.88	0.68	1.03	1.59
Exponentially Increasing Percentages	0.78	1.23	2.00	0.65	1.01	1.63
Equivalent Annuity Percentages	0.77	1.23	1.97	0.64	1.00	1.58
Canadian RRIF	0.80	1.20	1.86	0.70	1.01	1.54
1/E(t) – All	0.79	1.21	1.90	0.68	1.04	1.62
1/E(t) - Annuitants	-	-	-	-	-	-

Table 15: Overall Return on Fund and Estimated Return for Pension Holder and Heirs based upon Annuitant Population Mortality Tables.

Withdrawal Strategy	Total MW and Bequest - Annuitants			MW + Bequest Taxed at 55% – Annuitants		
	10th Percentile	Median	90th Percentile	10th Percentile	Median	90th Percentile
Annuity	0.93	0.93	0.93	0.93	0.93	0.93
Fixed Value 4.66p per £1	0.75	1.23	2.43	0.70	1.05	1.61
Fixed Value 5.59p per £1	0.75	1.17	2.25	0.72	1.06	1.63
Fixed Percentage 4.66%	0.77	1.31	2.31	0.62	1.02	1.72
Fixed Percentage 5.59%	0.78	1.27	2.23	0.66	1.03	1.74
Linearly Increasing Percentages	0.78	1.21	1.94	0.71	1.09	1.73
Exponentially Increasing Percentages	0.77	1.25	2.08	0.69	1.09	1.80
Equivalent Annuity Percentages	0.77	1.25	2.06	0.67	1.07	1.74
Canadian RRIF	0.80	1.21	1.92	0.72	1.08	1.68
1/E(t) – All	-	-	-	-	-	-
1/E(t) - Annuitants	0.78	1.25	2.10	0.69	1.10	1.82

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